

# OPTIMIZATION

Math 130 - Essentials of Calculus

7 April 2021

# STARTING EXAMPLE

## EXAMPLE

*A farmer has 2400ft of fencing and wants to fence off a rectangular field that borders a straight river. She needs no fence along the river. What are the dimensions of the field that has the largest area?*

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- 5 Determine the desired maximum or minimum value using calculus.

## NOW YOU TRY IT!

## EXAMPLE

*Find the dimensions of a rectangle with perimeter 100m whose area is as large as possible.*



## EXAMPLE WITHOUT A FEASIBLE DOMAIN

### EXAMPLE

*Find the dimensions of a rectangle with area  $1000\text{m}^2$  whose perimeter is as small as possible.*

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## EXAMPLE

*A box with a square base and open top must have a volume of  $32,000\text{cm}^3$ . Find the dimensions of the box that minimize the amount of material used.*

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- 1 Find two numbers whose difference is 100 and whose product is a maximum.
- 2 Find two positive numbers whose product is 100 and whose sum is a minimum.
- 3 Find a positive number such that the sum of the number and its reciprocal is as small as possible.